# End of Semester Assessment

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# Section A

## Consider a component that would experience significant error in applying one of the analytical techniques in this course (A1)

### Consider a component in the cutaway relevant regarding shear buckling (A2)

169: *Leading edge rib structure*

Shear buckling would be relevant in the webs of the rib structure. The ribs primarily transfer forces from skin panels to the spars and stringers. They are generally under compression. The concentration of the forces at the various contact points on the rib could vary along its length and will be large at the point where the force is transferred to the stringers/spars. An uneven force at across these contact points would lead to shear stress in the rib's web and consequently could create the conditions for shear buckling to occur. Most sides of the rib would have fixed boundary conditions as they connect to the wing panels or further supports, of note is the hole in the web panel. This would act to reduce the webs resistance to shear buckling as the boundary here would be a combination of simply supported and fixed.

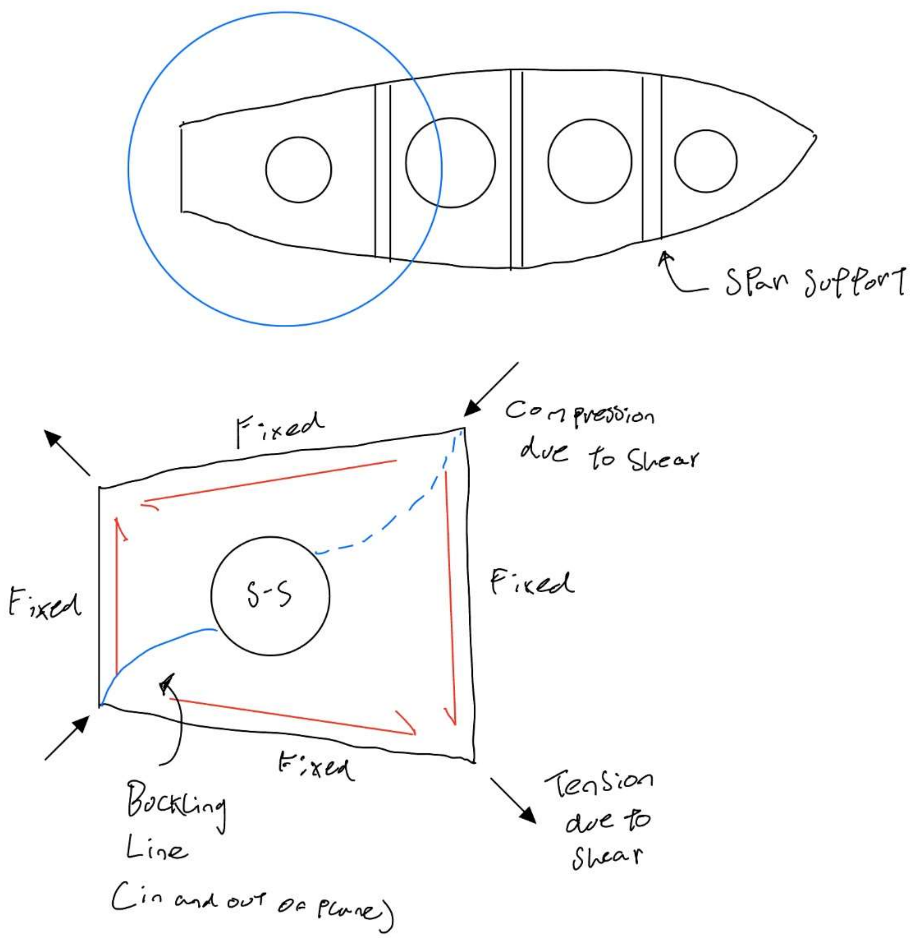


*Figure 1:Rib Web structure and the subsequent shear buckling response.*

## Consider a component in the cutaway experiencing bending (A3)

147: *Outer wing panel rib and stringer structure*

Specifically, the stringers, they will carry the bending induced by the wings lift and/or gravity. These forces will induce a bending moment along the stringer.

The stringers are relatively short and are connected to ribs periodically. It makes sense therefore to consider the stringers bending response as though it is broken into multiple stringers with lengths equal to the distance between the ribs. Given any stringer the loading in flight will act to create a bending moment that will induce compression at the top of the stringer and tension at the bottom.

Changing the cross section will affect the Ix, Iy and Ixy properties. As the stringers will experience the highest magnitude contribution to the bending moment in the Y direction the Ix cross sectional property will have the largest influence on the bending response or in other words increasing the distance between the X neutral axis and the area in the Y direction will reduce the bending stress response experienced by the stringer.

# Section B

The Positive X direction is to the left and the positive Y direction is taken to be down. The coordinate systems origin is at Point 3 on the cross section.

## Calculating the centroid location and Ix, Iy, Ixy at the centroid (B1 a)

Below are the values and intermediary results used during the calculation of the centroid coordinates and Second Moment of Inertia Properties.

*Table 1: Areas and Locations of points in the coordinate system.*

| Point | Area [mm^2] | x* [mm] | y* [mm] |
|---|---|---|---|
| 1 | 790 | 770 | 0 |
| 2 | 310 | 220 | 0 |
| 3 | 200 | 0 | 0 |
| 4 | 790 | 770 | 170 |
| 5 | 310 | 220 | 420 |
| 6 | 200 | 0 | 420 |

*Table 2: Calculating Ax*, Ay*, and Second Moment of Area values for points assuming rectangular cross sections.*

| Point | Ax* | Ay* | Ix0 (Rect) | Iy0 (Rect) |
|---|---|---|---|---|
| 1 | 608300 | 0 | 65.83 | 41086583.33 |
| 2 | 68200 | 0 | 25.83 | 2482583.33 |
| 3 | 0 | 0 | 16.67 | 666666.67 |
| 4 | 608300 | 134300 | 65.83 | 41086583.33 |
| 5 | 68200 | 130200 | 25.83 | 2482583.33 |
| 6 | 0 | 84000 | 16.67 | 666666.67 |

*Table 3: Relevant totals for the above table.*

| Total Area [mm^2] | Total Ax* | Total Ay* |
|---|---|---|
| 2600 | 1353000 | 348500 |

*Table 4: Centroid Position.*

| Centroid Point | Value [mm] |
|---|---|
| x-bar | 520.385 |
| y-bar | 134.038 |

*Table 5: Cross Section Points based on Centroid Coordinate system. Ay^2 and Ax^2 Calculations.*

| Point | x [mm] | y [mm] | Ay^2 | Ax^2 |
|---|---|---|---|---|
| 1 | 249.615 | -134.038 | 14193384.25 | 49223193.79 |
| 2 | -300.385 | -134.038 | 5569555.84 | 27971584.32 |
| 3 | -520.385 | -134.038 | 3593261.83 | 54160029.59 |
| 4 | 249.615 | 35.962 | 1021653.48 | 49223193.79 |
| 5 | -300.385 | 285.962 | 25349940.46 | 27971584.32 |
| 6 | -520.385 | 285.962 | 16354800.30 | 54160029.59 |

*Table 6: Finding the cross sections total Ix, Iy and Ixy using the parallel axis theorem.*

| Point | Ix0 + Ay^2 | Iy0 + Ax^2 | Axy | Ixy0 (Rect) | Ixy0 + Axy |
|---|---|---|---|---|---|
| 1 | 14193450.08 | 90309777.12 | -26431869.08 | 0 | -26431869.08 |
| 2 | 5569581.68 | 30454167.65 | 12481558.43 | 0 | 12481558.43 |
| 3 | 3593278.50 | 54826696.25 | 13950310.65 | 0 | 13950310.65 |
| 4 | 1021719.31 | 90309777.12 | 7091477.07 | 0 | 7091477.07 |
| 5 | 25349966.29 | 30454167.65 | -26628518.49 | 0 | -26628518.49 |
| 6 | 16354816.96 | 54826696.25 | -29761997.04 | 0 | -29761997.04 |

*Table 7: Summary of Required Values. Iy, Ix and Ixy are the totals of their respective columns.*

| Properties | Value | Units |
|---|---|---|
| x-bar | 520.385 | mm |
| y-bar | 134.038 | mm |
| Iy | 351181282.1 | mm^4 |
| Ix | 66082812.82 | mm^4 |
| Ixy | -49299038.5 | mm^4 |

## Finding the direct stresses in each of stiffener and the minimum margin of safety in bending (B1 b)

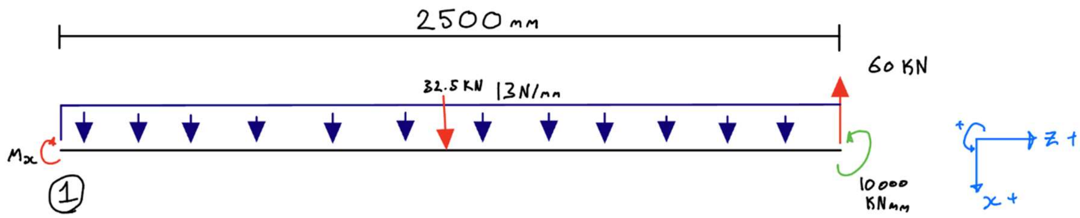


*Figure 2: Diagram of the beam used to find Mx*

$$\sum M_{point\ 1} = \ 0 = \ -M_x - (32.5 \cdot 1250) + (60 \cdot 2500) + 10000$$

$$M_x = 119375\ kN\ mm$$

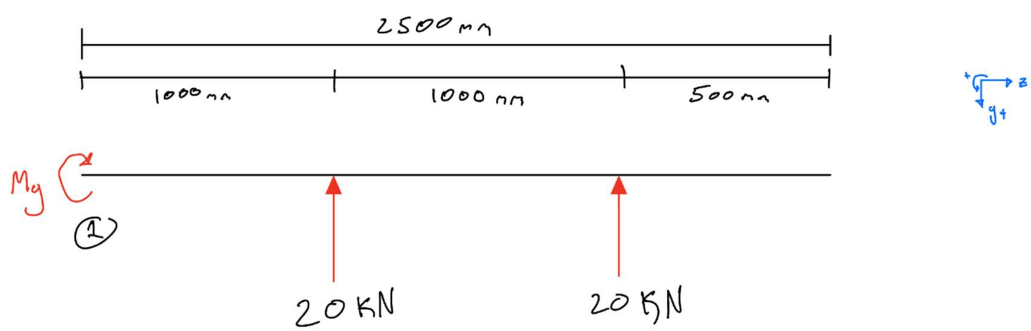


*Figure 3: Diagram of the beam used to find My*

$$\sum M_{point\ 1} = \ 0 = \ -M_y + (20 \cdot 1000) + (20 \cdot 2000)$$

$$M_y = 22020\ kN\ mm$$

Both moments will induce tension in the positive quadrant and so both values are taken to be positive for the bending equation.

$$\sigma_z = \left(\frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2}\right) x + \left(\frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2}\right) y$$

$$\sigma_z = \left(\frac{22020 \times 66082812.821 - 119375 \times -49299038.462}{351181282.051 \times 66082812.821 + 49299038.462^2}\right) x + \left(\frac{119375 \times 351181282.051 - 22020 \times \ -49299038.462}{351181282.051 \times 66082812.821 + 49299038.462^2}\right) y$$

$$\sigma_z = 0.0003532915866x + 0.002070007763y$$

*Table 8: Direct Stress on each stiffener. Calculated from the above bending stress equation. Negative is compression. Positive is tension. σcy is 230 MPa and σty is 245 MPa*

| Location | X [mm] | Y [mm] | Stress-Z [MPa] | Margin of Safety |
|---|---|---|---|---|
| 1 | 424.3461538 | -190.769231 | -244.9758626 | -0.0611 |
| 2 | -125.653846 | -190.769231 | -439.2862352 | -0.4764 |
| 3 | -345.653846 | -190.769231 | -517.0103843 | -0.5551 |
| 4 | -345.653846 | 229.2307692 | 352.392876 | 0.6952 |
| 5 | -125.653846 | 229.2307692 | 430.1170251 | 0.5696 |
| 6 | 424.3461538 | -20.7692308 | 106.925457 | 2.2913 |

## Calculate the in-plane shear flow in each panel and the minimum margin of safety in shear (B1 c)

Find shear force in the x and y directions.

$$\sum F_x = 0 = S_x + 32.5 - 60$$

$$S_x = 27.5\ kN$$

$$\sum F_y = 0 = S_y - 20 - 20$$

$$S_y = 40\ kN$$

Use asymmetric shear stress equation.

$$q_n = -\left(\frac{S_x I_x - S_y I_{xy}}{I_x I_y - I_{xy}^2}\right) A_n x_n - \left(\frac{S_y I_y - S_x I_{xy}}{I_x I_y - I_{xy}^2}\right) A_n y_n$$

$$q_n = -\left(\frac{27500 \times 66082812.821 - 40000 \times -49299038.462}{351181282.051 \times 66082812.821 + 49299038.462^2}\right) A_n x_n - \left(\frac{40000 \times 351181282.051 - 27500 \times -49299038.462}{351181282.051 \times 66082812.821 + 49299038.462^2}\right) A_n y_n$$

$$q_n = -0.0001823796702 \cdot A_n x_n - 0.0007413598224 \cdot A_n y_n$$

*Table 9: Find q basic using the above asymmetric stress equation. Positive is CCW around the cross section. Panel 5-6 has been cut.*

| Panel | Qb [N/mm] | An [mm^2] | Xn [mm] | Yn [mm] | Q0 [N/mm] |
|---|---|---|---|---|---|
| 6-3 | -23.419 | 200 | -520.385 | 285.962 | 0.00 |
| 3-2 | 15.437 | 200 | -520.385 | -134.038 | -23.42 |
| 2-1 | 63.225 | 310 | -300.385 | -134.038 | 15.44 |
| 1-4 | 105.763 | 790 | 249.615 | -134.038 | 63.23 |
| 4-5 | 48.737 | 790 | 249.615 | 35.962 | 105.76 |

*Table 10: Finding the total enclosed area for the cross section.*

| Shape | Base [mm] | Height [mm] | AEn [mm^2] |
|---|---|---|---|
| Horizontal Rect | 770 | 170 | 130900 |
| Vertical Rect | 220 | 250 | 55000 |
| Triangle | 550 | 250 | 68750 |
| Total | | | 254650 |

Need to use torsional equivalence to find q closing

*Table 11: External Torsion about point 5. CCW is Positive.*

| Property | X [mm] | Y [mm] | Total |
|---|---|---|---|
| *Distance from Centroid to Point 5* | 300.385 | 285.962 | |
| *Ext Force* | 27500 | 40000 | |
| *Torque [N mm]* | 8260588 | 11438480 | *19699067.5* |

*Table 12: Internal Torsion contribution from q basic about point 5. Using ql * h.*

| Panel | qb [N/mm] | Len [mm] | Perp Distance (Point 5) | Moment [N mm] |
|---|---|---|---|---|
| *6-3* | -23.4185 | 420 | 220 | -2163875.329 |
| *3-2* | 15.4370 | 220 | 420 | 1426387.74 |
| *2-1* | 63.2250 | 550 | 420 | 14604992.94 |
| *1-4* | 105.7633 | 170 | 550 | 9888876.488 |
| *4-5* | 48.7370 | 604.15 | 0 | 0 |
| *Total* | | | | *23756381.84* |

Find q closing from torsional equivalence.

$$T_{ext} = T_{int} = \sum q_b l \cdot h + 2 q_c A_e$$

$$q_c = \frac{T_{ext} - T_{qba}}{2 \cdot A_e}$$

$$q_c = \frac{19699067.5 - 23756381.84}{2 \cdot 254650}$$

$$q_c = -7.9665\ N/mm$$

*Table 13: Total shear flow in each panel. Negative is a CW flow about the cross section. Positive is CCW flow about the cross section.*

| Panel | qb + qc [N/mm] |
|---|---|
| *6-3* | -31.38501682 |
| *3-2* | 7.470644098 |
| *2-1* | 55.258625 |
| *1-4* | 97.79693225 |
| *4-5* | 40.77061532 |
| *5-6* | -7.966452652 |

## Calculate the bucking stress of the flexural and local modes in compression. Consider plasticity throughout. (B2 a)

Finding minimum principal axis.

$$I_{min}, I_{max} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x + I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{min} = 57798763.700 \text{ mm}^4$$

$$I_{max} = 359465331.172 \text{ mm}^4$$

Consider elastic buckling using Euler elastic equation.

*Table 14: Properties of the beam relevant to buckling.*

| Property | Value | Units |
|---|---|---|
| *L* | 5000 | mm |
| *L' (2 x L, fixed free setup)* | 10000 | mm |
| *Cross section area* | 2600 | mm^2 |
| *ρ (Radius of Gyration)* ($\sqrt{I_{min}/A}$) | 149.098 | mm |
| *Slenderness Ratio (L' / ρ)* | 67.070 | |
| *E (Youngs Modulus)* | 71000 | MPa |
| *Yield Stress* | 230 | MPa |

$$\sigma_{CR} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2}$$

$$\sigma_{CR} = \frac{\pi^2 \cdot 71000}{\left(\frac{10000}{149.098}\right)^2}$$

$$\sigma_{CR} = 156\ MPa$$

156 MPa is less than the yield stress 230 MPa so the column Is long and flexural buckling failure will occur in the elastic region.

Considering local buckling.

$$\sigma_{CR\ Local} = KE\left(\frac{t}{b}\right)^2$$

*Table 15: All a/b values indicate k∞ for boundary condition should be used. K∞ for Fixed-Fixed is 6.31. Critical Local Buckling Stress is calculated based on the given values.*

| Panel | t [mm] (Panel Thickness) | b [mm] (Panel Length) | a/b | Boundary Conditions | K Value | Buckling Stress [MPa] |
|---|---|---|---|---|---|---|
| 1 - 2 | 1.1719 | 550 | 9.091 | Fixed-Fixed | 6.31 | 2 |
| 2 - 3 | 1.1719 | 220 | 22.727 | Fixed-Fixed | 6.31 | 13 |
| 3 - 6 | 2.5 | 420 | 11.905 | Fixed-Fixed | 6.31 | 16 |
| 6 - 5 | 1.1719 | 220 | 22.727 | Fixed-Fixed | 6.31 | 13 |
| 5 - 4 | 1.1719 | 604.15 | 8.276 | Fixed-Fixed | 6.31 | 2 |
| 4 - 1 | 2.5 | 170 | 29.412 | Fixed-Fixed | 6.31 | 97 |

All critical local buckling stresses are bellow the Yield Stress of 230 MPa and so local buckling failure will occur in the elastic region.

Panel 1-2 and 5-4 will fail in local buckling at 2 MPa and is less than the flexural failure stress of 156 MPa.

Therefore, the beam will fail locally in the elastic region at 2MPa or an applied in plane force of 5.2 kN.

## Find the maximum axial force offset if the maximum allowable out-of-plane deflection is 10mm and the axial force is 1200 kN (B2 b)

*Table 16: Relevant values used to calculate eccentricity/axial force offset.*

| Property | Value | Units |
|---|---|---|
| P Crit | 405.0202 | kN |
| P | 1200 | kN |
| Allowable Deflection | 10 | mm |
| L' | 10000 | mm |
| Max y is at half L' | 5000 | mm |

$$\delta = \frac{4e}{\pi}\frac{1}{1-\left(\frac{P}{P_{cr}}\right)}$$

$$y(z) = \delta \sin\frac{\pi z}{L}$$

Rearranging to solve for e

$$e = \frac{y\left(1-\frac{P}{P_{cr}}\right)\pi}{4\sin\frac{\pi z}{L}}$$

$$e = \frac{10 * \left(1-\frac{1200}{405.02}\right)\pi}{4\sin\pi\cdot\frac{5000}{10000}}$$

$$e\ (axial\ offset) = 15.41\ mm$$

## Calculate and sketch the in-plane shear stress distribution (B3 a)

*Table 17: Properties relevant to shear stress and torsion analysis.*

| Property | Value | Units |
|---|---|---|
| *Torsion* | 25000 | kN mm |
| *AE* | 254650 | mm^2 |
| *G* | 27000 | MPa |
| *L* | 5000 | mm |
| *L/2* | 2500 | mm |

Find q due to torsion.

$$q = \frac{T}{2A_e}$$

$$q = \frac{25000}{2 \cdot 254650}$$

$$q = 0.0491\ kN/mm$$

*Table 18: Calculating s/t and the shear stress in each panel.*

| Panel | t (Panel Thickness) [mm] | s (Panel Length) [mm] | s/t | Shear Stress (q x t x 1000) [N] |
|---|---|---|---|---|
| *1 - 2* | 1.1719 | 550 | 644.545 | 41.8866645 |
| *2 - 3* | 1.1719 | 220 | 257.818 | 41.8866645 |
| *3 - 6* | 2.5 | 420 | 1050 | 19.63479285 |
| *6 - 5* | 1.1719 | 220 | 257.818 | 41.8866645 |
| *5 - 4* | 1.1719 | 604.15 | 708.0034 | 41.8866645 |
| *4 - 1* | 2.5 | 170 | 425 | 19.63479285 |

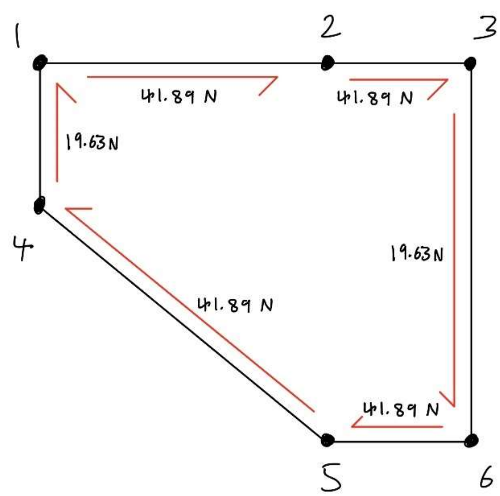


*Figure 4: Sketch of the stress distribution throughout the cross section.*

## Calculate the rate of twist (B3 b)

Find J for a closed section.

$$J = \frac{4A_e^2}{\sum(\frac{s}{t})}$$

$$J = \frac{4 \cdot 54650\text{^}2}{(644.54) + (257.82) + (1050) + (257.82) + (708) + (19.63)}$$

$$J = 77586653.96\ mm^4$$

Find Rate of Twist.

$$\frac{d\theta}{dz} = \frac{T}{GJ}$$

$$\frac{d\theta}{dz} = \frac{25000000}{27000 \cdot 77586653.96}$$

$$\frac{d\theta}{dz} = 1.1 \times 10^{-5}\ rad/mm$$

## Does this beam undergo warping? Why/why not? (B3 c)

Yes, the beam does undergo warping. We assume the beam is unrestrained which would indicate a consistent warping profile along the length of the beam and will allow for the use of elementary torsion theory for analysis.

The warping is the result of the shear stress acting on a non-circular profile which is what is being considered in this assessment, the external torsion induces this shear stress which is constant along the beams length at any given cross section provided the beam is considered unrestrained. This shear stress acts to deform the cross section when acting on a non-circular section as it acts parallel to the edges creating a rotation and inducing the warping effect.

If the beam were instead restrained the warping profile would vary along the length of the beam due to the added stress from the support acting to resist the torsion/shear stress. So, the warping profile would vary in comparison to the unrestrained situation, but warping would still occur.

So, either way given a non-circular cross section the beam will undergo warping.

## Calculate the torsion constant of the cross-section if the panel between stiffener 1 and stiffener 2 is removed (B3 d)

Section with out the 1-2 Panel will now be open instead of closed. Finding J for an open section.

$$J = \sum \frac{st^3}{3}$$

| Panel | t (Panel Thickness) [mm] | s (Panel Length) [mm] | J [mm^4] |
|---|---|---|---|
| 2 - 3 | 1.1719 | 220 | 118.0247 |
| 3 - 6 | 2.5 | 420 | 2187.5 |
| 6 - 5 | 1.1719 | 220 | 118.0247 |
| 5 - 4 | 1.1719 | 604.15 | 324.1121 |
| 4 - 1 | 2.5 | 170 | 885.4167 |
| *Total* | | | *3633.08* |

So, J (Torsion constant) for the new section is 3633.08 mm^4